New Jersey High School Algebra II Course
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Introduction

Algebra is the language through which much of mathematics is communicated and is therefore fundamental for both further education in mathematics and effective use of mathematics in the workplace. Students need to understand how quantities are related to one another and how algebra can be used to concisely express and analyze those relationships. Modern technology provides tools for supplementing the traditional focus on algebraic procedures, such as solving equations, with a more visual perspective, with graphs of equations displayed on a screen. Students can then focus on understanding the relationship between the equation(s) and the graph, and on what the graph represents in a real-life situation.

Algebra II extends the fundamental concepts and skills of elementary algebra to a higher level, and, while introducing new concepts, draws upon the same basic themes studied previously. Algebra II also offers the opportunity to apply algebraic skills and reasoning to the related mathematical areas of trigonometry, data analysis, probability, and discrete mathematics. Districts throughout the State have requested a "model" or "sample" of how some of this non-traditional (less than 100 years old) content might mesh with the traditional (more than 100 years old) algebra I, geometry, algebra II (or algebra I, algebra II, geometry) course sequence.

This document provides more detail than is included in the more limited and concise K-12 Core Curriculum Content Standards. *Italicized clarification* of algebra II content builds on and extends the core content. Some of the clarifying italicized language is adapted from Achieve, Inc. core course content drafts or the Mathematics Benchmarks from "Ready or Not: Creating a High School Diploma that Counts" (Achieve, 2004). Clarifying examples were adapted not only from the Achieve documents, but also from New Jersey's 1989 Core Course Proficiencies and from the "New Jersey Mathematics Curriculum Framework" (1996).

Algebra has been referred to as a "gatekeeper" for the future study of mathematics, science, the social sciences, business, and a host of other areas. In the past, algebra has served as a filter, screening people out of these opportunities. For New Jersey to be part of the global society, it is important that algebra play a major role in a mathematics program that opens the gates for all students.

New Jersey High School Algebra II Course
DRAFT

Interpreting the Document

The New Jersey Core Curriculum Content Standards and Cumulative Progress Indicators (CPIs) are highlighted with bold text and are further identified by their alphanumeric code. For Example, **4.3.12 C 3** refers to:

- Mathematics (**4**)
- **Patterns and Algebra** (4.**3**)
- by the end of grade **12** (4.3.**12**)
- Strand **C**: **Modeling** (4.3.12 **C**)
- Cumulative Progress Indicator (CPI): **Students will convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling).** (4.3.12 C **3**)

It is important to note that "grade **12**" is not used to code content which students are expected to learn during their senior year in high school. Rather, it refers to knowledge and skills that will generally be learned during the 9-12 (high school) years or, in some cases, even earlier. As this applies to the study of algebra, not just algebra II, but also many of the algebra I cumulative progress indicators, are coded with a "12."

The Cumulative Progress Indicators associated with Standard 4.2 (Geometry and Measurement) are not included in this document, since they would be achieved in a geometry course or previously. For the sake of completeness, all other high school CPIs are included. Several of them are labeled as having been achieved in algebra I or geometry and may have been achieved in even earlier grades.

Each strand begins with a description of the strand (Descriptive Statement), taken from the Core Curriculum Content Standards for Mathematics.

Standard 4.5 (Mathematical Processes), which applies across all grade levels and all mathematics courses, has been included as Appendix A for easy reference.

New Jersey High School Algebra II Course
DRAFT

4.1.12 A. Number Sense

Descriptive Statement: Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.

4.1.12 A 1. Students will extend understanding of the number system to all real numbers *is assumed to have been achieved in algebra I. In algebra II, it is anticipated that students will further extend their understanding of the number system to imaginary and complex numbers (See particularly* ***4.1.12 A 3*** *and* ***4.1.12 B 1*** *below).*[1]

4.1.12 A 2. Students will compare and order rational and irrational numbers *is assumed to have been achieved in algebra I.*

4.1.12 A 3. Develop conjectures and informal proofs of properties of number systems and sets of numbers.

- *Know that the decimal expansion of an irrational number is non-terminating and non-repeating.*
- *Show that every interval on the real number line, no matter how small, contains rational numbers.*
- *Given a degree of precision, determine a rational approximation for an irrational number.*
  - *Example: Given a square table top that measures 1 yard on each side, calculate the length of the diagonal to the nearest sixteenth of an inch.*
- *Understand and explain why every real number is also a complex number. (It follows that every real number, a, is a complex number because it can be expressed as a + 0i.)*

4.1.12 B. Numerical Operations

Descriptive Statement: Numerical operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand

[1] On this page and throughout the document, normal print is used for language taken directly from the current version of New Jersey's Core Curriculum Content Standards in Mathematics. *Italicized print* is used for clarifying language which builds on and extends the core content, linking it with a model algebra II course.

how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.

4.1.12 B 1. Extend understanding and use of operations to real numbers and algebraic procedures *is assumed to have been achieved in algebra I. Many students may also have been introduced to complex numbers in algebra I. In algebra II, it is anticipated that students will*:

- *Perform operations on numbers expressed in radical form.*
- *Add, subtract, and multiply complex numbers using the rules of arithmetic.*
- *Express the principal square root of a negative number in the form bi, where b is real (e.g.,* $\sqrt{-5} = i\sqrt{5}$ *;* $\sqrt{-225} = 15i$*).*
- *Recognize and use complex conjugates (The conjugate of a complex number* $a + bi$ *is the number* $a - bi$*).*
- *Know that complex solutions of quadratic equations with real coefficients occur in conjugate pairs, and use the rules of arithmetic to show this.*
- *Use conjugates to divide complex numbers.*

  Example:

$$\frac{(5 + i\sqrt{2})}{3 - i\sqrt{2}} = \frac{(5 + i\sqrt{2})}{(3 - i\sqrt{2})} \cdot \frac{(3 + i\sqrt{2})}{(3 + i\sqrt{2})} = \frac{15 + 8i\sqrt{2} + 2i^2}{9 - 2i^2} = \frac{13 + 8i\sqrt{2}}{11}$$

4.1.12 B 2. Develop, apply, and explain methods for solving problems involving rational and negative exponents.

4.1.12 B 3. Perform operations on matrices. *Students are assumed to know and to fluently use matrix notation for rows, columns, and entries of cells.*

- Addition and subtraction *(assumed to have been achieved in algebra I).*
- Scalar multiplication *(assumed to have been achieved in algebra I).*

In algebra II:

- *Multiplication of matrices.*
- *Additive identity matrix.*
- *Multiplicative identity matrix.*

4.1.12 B 4. Understand and apply the laws of exponents to simplify expressions involving numbers raised to powers.

- *Apply the laws of exponents to expressions containing rational exponents.*

4.1.12 C. Estimation

Descriptive Statement: Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.

4.1.12 C 1. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits. *In algebra II, as in earlier courses, this should not be done in isolation. Rather, it should be applied to particular problems (e.g., estimating the roots of an equation graphically).*

4.3.12 A. Patterns

Descriptive Statement: Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and, by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena**.**

4.3.12 A 1. Use models and algebraic formulas to represent and analyze sequences and series *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Explicit formulas for nth terms
- Sums of finite arithmetic series
- Sums of finite and infinite geometric series

4.3.12 A 2. Develop an informal notion of limit.

4.3.12 A 3. Use inductive reasoning to form generalizations *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

4.3.12 B. Functions and Relationships

Descriptive Statement: The function concept is one of the most fundamental unifying ideas of modern mathematics. Students begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations that express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.

4.3.12 B 1. Understand relations and functions and select, convert flexibly among, and use various representations for them, including equations or inequalities, tables, and graphs *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II. In algebra II, students will:*

- *Identify the conditions under which a function has an inverse.*
- *Determine whether two given functions are inverses of each other.*
- *Explain why the graph of a function and its inverse, when it exists, are reflections of one another over the line $y = x$.*
- *Determine the inverse of linear and simple non-linear functions, including any necessary restrictions on the domain.*

  Example: Show that the inverse of an exponential function is a logarithmic function and vice versa.

It would be appropriate for all students to have at least a definitional awareness of the relationship between logarithms and exponents. A cursory, technology-based exploration of logarithmic functions would be appropriate for all students in an algebra II course. Such an exploration would arise naturally from the study of exponential functions. Realistically, however, with the increasing availability of calculator technology over the past thirty-five years, logarithms have had decreasing computational utility for the average person. Even the slide rule, commonplace as recently as the 1960's, and largely based on logarithmic relationships, is now infrequently used in the workplace and in tertiary technical studies. Nevertheless, some school districts may choose to move a more intensive study of logarithmic functions from pre-calculus to algebra II, where it routinely appeared forty or fifty years ago. That should only be done if it is possible for the enrolled students to first learn the necessary trigonometry, data analysis, probability, and discrete mathematics delineated as important for ALL students in the Core Curriculum Content Standards for Mathematics. For a district that feels strongly that logarithms should be included in their algebra II course, the following is suggested as a reasonable expectation for all students:

- *Students will understand the relationship of logarithms to exponents. This understanding should be at the numerical level.*

  Examples:

  $\log_2 32 = 5$ *means the same thing as* $2^5 = 32$

  If $x = \log_{10} 3$, *then* $10^x = 3$ *(technology could then be used to find the value of x)*

  Explain why the number of digits in the binary representation of a decimal number N is approximately the logarithm to base 2 of N.

4.3.12 B 2. Analyze and explain the general properties and behavior of functions of one variable, using appropriate graphing technologies *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Slope of a line or curve
- Domain and range
- Intercepts

- Continuity
- Maximum/minimum
- Estimating roots of equations
- Intersecting points as solutions of systems of equations (*In algebra II, students will use both algebraic and graphing techniques to solve systems of equations. See the last bullet under CPI* **4.3.12 D 2** *in this document.*)
- Rates of change

In algebra II*, students will:*

- *Solve systems of linear inequalities in two variables using graphing techniques*
- *Use systems of equations and inequalities to represent situations (e.g., optimization problems, mixture problems)*

4.3.12 B 3. Understand and perform transformations on commonly-used functions *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Translations, reflections, dilations (*achieved in algebra I and geometry*)
- Effects on linear and quadratic graphs of parameter changes in equations (*In algebra II, students will identify the effect that changes in the coefficients of the quadratic function have on the shape and position of the graph.)*
- Using graphing calculators or computers for more complex functions

4.3.12 B 4. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, and trigonometric functions.

- Linear vs. non-linear. *In algebra II, distinguish among linear, exponential, polynomial, rational, power, or root functions by their symbolic form.*

  <u>*Example*</u>*: $f(x) = 3^x$ is an exponential function because the variable is in the exponent while $f(x) = x^3$ has the variable in the position of a base and is a power function.*
- Symmetry
- Increasing/decreasing on an interval

Students are assumed to have studied the key characteristics of quadratic functions and their graphs not only in algebra I, but especially in geometry (coordinate geometry occupying a large portion of the geometry curriculum). That is, a student who enters the algebra II course after taking a high school geometry course should already be thoroughly familiar with the vertex form of a quadratic function, $f(x) = a(x – h)^2 + k$, and be able to identify the vertex and axis of symmetry of the function's parabolic graph. For students who study algebra II before geometry, it would be appropriate for them to do this in algebra II. In algebra II, for polynomial functions in general, students will:

- *Know the general form for polynomial functions of degree n: $P(x) = ax^n + bx^{n-1} + \dots + px^2 + qx + r$ for n an integer, $n \geq 0$ and $a \neq 0$.*
- *Describe the effect that changes in the coefficients of $f(x) = ax^2 + bx + c$ have on the shape, position, and characteristics of the graph of f(x).*

- *Determine key characteristics of polynomial functions and their graphs (including why every polynomial function of odd degree has at least one zero in the set of real numbers).*
- *Understand the fundamental theorem of algebra (Every polynomial function with complex coefficients has at least one zero in the set of complex numbers) and the corollary that every polynomial function of degree n with complex coefficients has exactly n complex zeros, counting multiplicities.*
- *Determine if a given graph or table of values suggests a simple polynomial (i.e., linear, quadratic, or cubic) function.*
- *Determine, where possible, the domain, range, intercepts, and end behavior of a polynomial function. (It is not always possible to determine the exact horizontal intercepts for a polynomial function.*
- *Recognize situations that can be modeled by power and polynomial functions; create and use models to answer questions about those situations (e.g., the relationships between the radius and area of a circle, the radius and volume of a sphere, the number of sides and the number of diagonals of a polygon, the profit earned by a company and the price of its product, or the height of a falling projectile and time).*
- *Algebra II students may explore the Rational Root Theorem (In a polynomial with integer coefficients, if a rational number c/d is reduced to its lowest terms and is a solution to the equation, then c is a factor of the constant term of the polynomial and d is a factor of the leading coefficient of the polynomial).*

***In algebra II**, for power functions, students will:*

- *Identify the characteristics of graphs of power functions of the form $f(x) = ax^n$ for both positive and negative integral values of n (including the symmetry of an odd or even power function) and know that even power functions have either a minimum or a maximum value while odd power functions have neither.*
- *Recognize that a power function with an exponent that is a positive integer is a particular type of polynomial function whose graph contains the origin.*

  <u>Example</u>: $f(x) = x^3 - 5$ is a polynomial function but not a power function because of the added constant.
- *Recognize that power functions are generalizations of the inverse proportional function $f(x) = k/x$ $[f(x) = kx^n$ for $n = -1]$ and the direct proportional function $f(x) = kx$ $[f(x) = kx^n$ for $n = 1]$ and apply the characteristics of power functions to these two special cases.*
- *Use properties of exponents and roots to transform the algebraic representation of power functions.*

  <u>Example</u>: $f(x) = 3x^2 (-2x^{(-3/2)})$ is more easily identified as a root function once it is rewritten as $f(x) = -6x^{(1/2)} = -6\sqrt{x}$.
- *Explain and illustrate the effect that a change in a parameter has on a power function [i.e., a change in a or n for $f(x) = ax^n$].*
- *Differentiate the graphs of simple exponential and power functions by their key characteristics. (Be aware that it can be very difficult to distinguish*

graphs of these types of functions over small regions or particular subsets of their domains. Sometimes the context will suggest a likely type of function.)

- *Use the laws of exponents to determine exact solutions for problems involving power functions where possible; otherwise, approximate the solutions graphically or numerically.*

***In algebra II**, for exponential functions, students will:*

- *Recognize the graphs of simple exponential functions.*
- *Determine key characteristics of exponential functions and their graphs (e.g., domain, range, intercepts, asymptotes, and end behavior).*
- *Identify the effect that changes in the parameters or base have on the graph of the exponential function* $f(x) = a \bullet b^x + c$.
- *Recognize and solve problems that can be modeled using exponential functions (e.g., growth and decay, or compound interest)*

  > *Example: If a culture had 500 cells at noon and 600 cells at 1 pm, what is the approximate doubling time of the cell population? Approximately how many cells will there be at 4 pm?*

***In algebra II**, for periodic functions, students will:*

- *Recognize periodic phenomena (e.g., sound waves, the height above the ground of a car on a Ferris wheel, or the length of time from sunrise to sunset over a 10-year period)*
- *Know that the sine and cosine functions are used to describe periodic behavior*
- *Investigate and identify key characteristics of periodic functions and their graphs (i.e., period, amplitude, maximum, and minimum)*

  > *Example: Explain the difference between frequency modulation used in FM radio signals and amplitude modulation used in AM radio signals.*

- *Use basic properties of frequency and amplitude to solve problems (e.g., identify the length of a cycle in a situation exhibiting periodic behavior in order to make predictions)*

***In algebra II**, for piecewise-defined functions, students will:*

- *Recognize and graph absolute value, step, and other piecewise-defined functions.*
- *Interpret the algebraic representation for a given piecewise-defined function*
- *Write an algebraic representation for a given piecewise-defined function*
- *Determine vertex, slope of each branch, intercepts, and end behavior of an absolute value graph*
- *Recognize and solve problems that can be modeled using absolute value, step, and other piecewise-defined functions (e.g., postage rates or taxi charges)*

4.3.12 C. Modeling

Descriptive Statement: Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).

4.3.12 C 1. Use functions to model real-world phenomena and solve problems that involve varying quantities *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Linear, quadratic, exponential, periodic (sine and cosine), and step functions (e.g., price of mailing a first-class letter over the past 200 years)
- Direct and inverse variation
- Absolute value
- Expressions, equations and inequalities
- Same function can model variety of phenomena
- Growth/decay and change in the natural world
- Applications in mathematics, biology, and economics (including compound interest)

Example: Given mass and total force, manipulate the equation representing gravitational force to find the distance between two objects in space.

***In algebra II**, students will:*

- *Graph quadratic inequalities and use them to answer questions*
- *Recognize situations that can be modeled by quadratic functions; create and use models to answer questions about those situations. (e.g., The relationship between the length of the side of a square and its area, or distance traveled vs. time for a falling object.)*
- *Recognize and solve problems that can be modeled using power and root functions (e.g., use power functions to represent quantities arising from geometric contexts such as length, area, and volume).*

4.3.12 C 2. Analyze and describe how a change in an independent variable leads to change in a dependent one. *In algebra II, students will:*

- *Recognize the relationship between $f(x)$ and $f(c\cdot x)$, and how that differs from $c\cdot f(x)$. Recognize the relationship between $f(x)$ and $f(c+x)$, and how that differs from $c+f(x)$. Students will apply that understanding to solving problems in situations which can be modeled by functions.*
- *Find the composition of two functions.*

Example: Find $g[f(x)]$, for $f(x) = x + 1$ and $g(x) = x^2$.

4.3.12 C 3. Convert recursive formulas to linear or exponential functions (e.g., Tower of Hanoi and doubling). *In algebra II, students will:*

- *Use and interpret relationships represented iteratively and recursively.*

- *Use iteration to demonstrate the effect of compound interest on loans or investments.*
  <u>*Examples*</u>*:*
  Use spreadsheets to display the amount of principal and interest paid for each payment in the life of a loan.
  Use iteration and past data about the yearly percent increase of college tuition and annual inflation rate to estimate the cost of college for a newborn in current dollar equivalents.
- *Use recursion to define the factorial function.*

4.3.12 D. Procedures

Descriptive Statement: Techniques for manipulating algebraic expressions – procedures – remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.

4.3.12 D 1. Evaluate and simplify expressions *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Add and subtract polynomials
- Multiply a polynomial by a monomial or binomial (*In algebra II, multiply polynomials.*)
- Divide a polynomial by a monomial (*In algebra II, divide a polynomial by a first- or second-degree polynomial.*)

In algebra II*, students will:*

- *Use factoring to simplify the quotient of two polynomials where possible.*
- *Use polynomial relations to solve problems (e.g., the number of sides and the number of diagonals of a polygon; the areas of simple plane figures and their linear dimensions; the surface areas of simple three-dimensional solids and their linear dimensions)*
- *Perform simple operations with rational expressions and algebraic fractions, including complex fractions. (Note that these expressions should be limited to linear and factorable quadratic denominators.) Rewrite complex fractions composed of simple fractions in lowest terms.*
- *Evaluate polynomial and rational expressions.*

4.3.12 D 2. Select and use appropriate methods to solve equations and inequalities *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II.*

- Linear equations – algebraically (*In algebra II, students will solve simple non-linear equations and inequalities in one variable.*)

- *Solve simple polynomial equations, and use technology to approximate solutions for more complex polynomial equations.*
- *Solve higher-degree polynomial and exponential equations graphically and numerically.*
- *Solve power equations with integer exponents, of the form $ax^n = b$, algebraically (when logarithms are not needed for solution), graphically, and using technology.*
- Quadratic equations – factoring (when the coefficient of x^2 is 1) and using the quadratic formula. *In algebra II, students will determine complex solutions of the form $a \pm bi$ for quadratic equations, and explain how complex numbers emerge as solutions to certain quadratic equations.*
- *Choose and use a method for solving a quadratic equation (e.g., factoring, completing the square, quadratic formula, or technology) that is appropriate for the context.*
- *Use the quadratic formula to prove that the abscissa of the vertex of the of the corresponding parabola is half way between the roots of the equation*
- *Relate the signs of the coefficients a, b, and c to the roots of the quadratic equation, $ax^2 + bx + c = 0$.*
- *Recognize that the nature of the roots of the equation $ax^2 + bx + c = 0$ depends on the discriminant, $D = b^2 – 4ac$. The roots are real and distinct if $D > 0$; real and equal if $D = 0$; and complex conjugates if $D < 0$.*
- *Solve formulas and other literal equations for a given variable*
  Example: Solve $A = (1/2)nr^2 + r^2$ for r.
- All types of equations using graphing, computer, and graphing calculator techniques. *In algebra II, for example, students might be asked to find the solutions to $2^x = 3x^2$. (Some students use a spreadsheet to develop a table of values. Once they find an interval of length 1 which contains a solution, they refine their numbers to develop the answer to the desired precision. Other students graph both $y = 2^x$ and $y = 3x^2$ using graphing calculators or computers and use the trace function to determine where they intersect. Other students graph the function $f(x) = 3x^2 – 2^x$ and use the trace function to find the zeros. Other students enter the function in their graphing calculators and check the table of values.)*
- *Solve simple rational equations in one variable.*
  Example: Solve for r: $\frac{2}{r-1} + r = 4$
- *Solve radical equations algebraically, graphically, and with calculators.*
  Example: Solve for X: $\sqrt{X-2} = 3$.
- *Know which operations on an equation produce an equation with the same solutions, and which produce an equation with fewer or more solutions (lost or extraneous roots, respectively)*
- *Graph simple rational and radical equations in two variables. Relate the algebraic properties of a rational or radical equation to the geometric properties of its graph.*
  Example: Given the equation $y = \frac{1}{\quad}$

$x^2 - 9$

determine algebraically, from the equation, that x may be any real number except 3 or -3. Relate these values to the location of the asymptotes on the graph. Further, discuss the relationship between the value of x and the value of the fraction in the equation on the one hand and the rise or fall of the curve on the other hand.

- *Solve linear inequalities algebraically.*
- *Solve equations and inequalities involving the absolute value of a linear expression.*
- *Solve and graph quadratic inequalities in one or two variables (Note that problems involving quadratic equations should not be limited to simple, factorable forms. Varied methods of solution including using the quadratic formula and approximating with calculator technology should be expected.)*
- *Solve systems of equations. In algebra II, students should have an opportunity to explore various methods for solving systems of equations in addition to substitution (e.g., graphing, determinants, Gaussian elimination, or addition and subtraction), including the use of calculator technology. In a given situation, a student could then choose the most appropriate method.*

  Examples:

  Solve for x, y, and z: $3x - 2y + z = 7$, $x + z = 1$, $y = 4x$.

  Solve for x and y: $x + y = 7$, $x^2 + y^2 = 25$.

4.3.12 D 3. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology. (*In algebra II, interpret an algebraic or graphical solution in terms of the context of the problem and using appropriate units of measurement.)*

4.4.12 A. Data Analysis

Descriptive Statement: In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.

4.4.12 A 1. Use surveys and sampling techniques to generate data and draw conclusions about large groups *is assumed to have been partially achieved in algebra I, but built upon and extended in algebra II*.

- Advantages/disadvantages of sample selection methods (e.g., convenience sampling, responses to survey, random sampling).

4.4.12 A 2. Evaluate the use of data in real-world contexts.

- Accuracy and reasonableness of conclusions drawn.

- Bias in conclusions drawn (e.g., influence of how data is displayed). *Students are assumed to have worked with frequency data (**4.4.8 A 1**) in middle school. In algebra II, students are expected to continue their use of frequency data and to especially identify idiosyncrasies and potential misuses of weighted averages.*

    Example: Suppose a company employed 100 women with average annual salaries of $20,000 and 500 men with average salaries of $40,000. After a change in management, they employed 200 women and 400 men. To correct past inequities, the new management increased women's salaries by 25% and men's salaries by 5%. Despite these increases, the company's average salary declined by almost 1%.
- Statistical claims based on sampling.

4.4.12 A 3. Design a statistical experiment, conduct the experiment, and interpret and communicate the outcome.

4.4.12 A 4. Estimate or determine lines of best fit (or curves of best fit if appropriate) with technology, and use them to interpolate within the range of the data.

4.4.12 A 5. Analyze data using technology, and use statistical terminology to describe conclusions.

- Measures of dispersion: variance, standard deviation, outliers
- Correlation coefficient
- Normal distribution (e.g., approximately 95% of the sample lies between two standard deviations on either side of the mean)

In algebra II*, students will:*

- *Understand that in the normal distribution approximately 2/3 of the data lie within one standard deviation of the mean.*
- *Understand common examples that fit the normal distribution (e.g., height, weight).*

4.4.12 B. Probability

Descriptive Statement: Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.

4.4.12 B 1. Calculate the expected value of a probability-based game, given the probabilities and payoffs of the various outcomes, and determine whether the game is fair *is assumed to have been achieved in algebra I.*

4.4.12 B 2. Use concepts and formulas of area to calculate geometric probabilities *is assumed to have been achieved in geometry.*

4.4.12 B 3. Model situations involving probability with simulations (using spinners, dice, calculators and computers) and theoretical models, and solve problems using these models *is assumed to have been achieved in algebra I. In algebra II, it is anticipated that students will*:

- *Use probability to interpret odds and risks.*
- *Recognize common misconceptions*

  <u>Example</u>: After a fair coin has come up four times in a row, explain why the probability of tails is still 50%.

4.4.12 B 4. Determine probabilities in complex situations *is assumed to have been partially achieved in algebra I or earlier, but built upon and extended in algebra II.*

- Conditional events *(Assumed to have been achieved and applied to grade-appropriate applications in middle school. In algebra II, students would be expected to encounter grade-appropriate applications which extend their knowledge and skills.)*
- Complementary events
- Dependent and independent events (*Clearly distinguish between dependent and independent events*).
- *Mutually exclusive events (Two events that have NO outcomes in common are called mutually exclusive. These are events that cannot occur at the same time.)*

  <u>Example</u>: A pair of dice is rolled. The events of rolling a 9 and of rolling a double have no outcomes in common. These two events are mutually exclusive.

  For any two mutually exclusive events, the probability that an outcome will be in one event or the other event is the sum of their individual probabilities-- P(A or B) = P(A) + P(B).
- *Non-mutually exclusive events*

  <u>Example</u>: A pair of dice is rolled. The events of rolling a 6 and of rolling a double have the outcome (3,3) in common. These two events are not mutually exclusive.

  For any two events which are not mutually exclusive, the probability that an outcome will be in one event or the other event is the sum of their individual probabilities minus the probability of the outcome being in both events-- P(A or B) = P(A) + P(B) - P(A and B).

4.4.12 B 5. Estimate probabilities and make predictions based on experimental and theoretical probabilities. *In algebra II, these should include compound events.*

4.4.12 B 6. Understand and use the "law of large numbers" (that experimental results tend to approach theoretical probabilities after a large number of trials).

4.4.12 C. Discrete Mathematics—Systematic Listing and Counting

Descriptive Statement: Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.

4.4.12 C 1. Calculate combinations with replacement (e.g., the number of possible ways of tossing a coin 5 times and getting 3 heads) and without replacement (e.g., number of possible delegations of 3 out of 23 students). F*amiliarity with both combinations and permutations (**4.4.7 C 1**), including the understanding and use of factorial notation (**4.4.8 C 1**), is assumed to have been achieved and applied to grade-appropriate tasks in middle school. In algebra II, students would be expected to encounter grade-appropriate applications which extend their knowledge and skills.*

- *Use combinations, permutations, and other systematic counting methods to determine the number of ways events can occur.*
- *Understand and use factorial notation.*

4.4.12 C 2. Apply the multiplication rule of counting in complex situations, recognize the difference between situations with replacement and without replacement, and recognize the difference between ordered and unordered counting situations.

<u>Examples</u>: *How many different four-digit numbers can be formed if the first digit must be non-zero and each digit may be used only once? How many are possible if the first digit must be non-zero but digits can be used any number of times?*

4.4.12 C 3. Justify solutions to counting problems. *Note particularly Cumulative Progress Indicators **4.5 D 1** and **4.5 D 2**.*

4.4.12 C 4. Recognize and explain relationships involving combinations and Pascal's Triangle, and apply those methods to situations involving probability. *In algebra II:*

- *Relate the expansion of $(a + b)^n$ with the possible outcomes of a binomial experiment and the n^{th} row of Pascal's triangle.*

4.4.12 D . Discrete Mathematics-Vertex-Edge Graphs and Algorithms

Descriptive Statement: Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.

4.4.12 D 1. Use vertex-edge graphs and algorithmic thinking to represent and solve practical problems *is assumed to have been achieved in geometry or previously. Students are assumed to know what an algorithm is and to be able to describe simple computational or algebraic algorithms.*

- Circuits that include every edge in a graph
- Circuits that include every vertex in a graph
- Scheduling problems (e.g., when project meetings should be scheduled to avoid conflicts) using graph coloring
- Applications to science (e.g., who-eats-whom graphs, genetic trees, molecular structures)

4.4.12 D 2. Explore strategies for making fair decisions *is assumed to have been achieved in geometry or previously.*

- Combining individual preferences into a group decision (e.g., determining winner of an election or selection process)
- Determining how many Student Council representatives each class (9^{th}, 10^{th}, 11^{th}, and 12^{th} grade) gets when the classes have unequal sizes (apportionment)

New Jersey High School Algebra II Course

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Appendix A

MATHEMATICAL PROCESSES

Standard 4.5. All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

4.5 A 1. Problem Solving

Descriptive Statement: Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 A 1. Learn mathematics through problem solving, inquiry, and discovery.

4.5 A 2. Solve problems that arise in mathematics and in other contexts.

- Open-ended problems
- Non-routine problems
- Problems with multiple solutions
- Problems that can be solved in several ways

4.5 A 3. Select and apply a variety of appropriate problem-solving strategies (e.g., “try a simpler problem” or “make a diagram”) to solve problems.

4.5 A 4. Pose problems of various types and levels of difficulty.

4.5 A 5. Monitor their progress and reflect on the process of their problem solving activity.

4.5 B. Communication

Descriptive Statement: Communication of mathematical ideas involves students’ sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 B 1. Use communication to organize and clarify their mathematical thinking.

- Reading and writing
- Discussion, listening, and questioning

4.5 B 2. Communicate their mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.

4.5 B 3. Analyze and evaluate the mathematical thinking and strategies of others.

4.5 B 4. Use the language of mathematics to express mathematical ideas precisely.

4.5 C. Connections

Descriptive Statement: Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 C 1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).
4.5 C 2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).
4.5 C 3. Recognize that mathematics is used in a variety of contexts outside of mathematics.
4.5 C 4. Apply mathematics in practical situations and in other disciplines.
4.5 C 5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).
4.5 C 6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

4.5 D. Reasoning

Descriptive Statement: Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.

In recent years, there has been heated debate concerning the intensity and formality of "proof" that should be included in high school mathematics courses, particularly geometry. This document will not resolve that debate. However, one thing is very clear—with whatever level of intensity and/or formality students learn to use reasoning and proof, that level should not be limited to only one course (e.g., geometry). Rather, students should have the opportunity to "select and use various types of reasoning and methods of proof" ***(CPI 4.5 D 3****) throughout their study of both algebra (algebra I and algebra II) and geometry. Students should not leave high school believing that reasoning and/or deductive proofs are a "geometry thing."*

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 D 1. Recognize that mathematical facts, procedures, and claims must be justified.
4.5 D 2. Use reasoning to support their mathematical conclusions and problem solutions.
4.5 D 3. Select and use various types of reasoning and methods of proof.
4.5 D 4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.
4.5 D 5. Make and investigate mathematical conjectures.

- Counterexamples as a means of disproving conjectures
- Verifying conjectures using informal reasoning or proofs.

4.5 D 6. Evaluate examples of mathematical reasoning and determine whether they are valid.

4.5 E. Representations

Descriptive Statement: Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 E 1. Create and use representations to organize, record, and communicate mathematical ideas.
- Concrete representations (e.g., base-ten blocks or algebra tiles)
- Pictorial representations (e.g., diagrams, charts, or tables)
- Symbolic representations (e.g., a formula)
- Graphical representations (e.g., a line graph)

4.5 E 2. Select, apply, and translate among mathematical representations to solve problems.
4.5 E 3. Use representations to model and interpret physical, social, and mathematical phenomena.

4.5 F. Technology

Descriptive Statement: Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).

At each grade level, with respect to content appropriate for that grade level, students will:

4.5 F 1. Use technology to gather, analyze, and communicate mathematical information.
4.5 F 2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.
4.5 F 3. Use graphing calculators and computer software to investigate properties of functions and their graphs.
4.5 F 4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).
4.5 F 5. Use computer software to make and verify conjectures about geometric objects.
4.5 F 6. Use computer-based laboratory technology for mathematical applications in the sciences.